

## A perturbed cognate mechanism as a force sensor

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### Abstract

The cognate system is an unique planar mechanism in which a stationary point is generated because of special dimensions. The resulting 10 link mechanism forms a structure which has one internal degree of freedom. This property is leveraged by using the stationary joint as a point of loading to obtain a force sensor with theoretically infinite stiffness, subject to the condition that the mechanism is able to exhibit different pose signatures with variable loading. This theoretical model of the cognate mechanism is perturbed and the kinematic and dynamic properties of this new class of mechanism is analysed to demonstrate that it is possible to synthesize a compliant linkage based force sensor that may be used for haptic feedback and other applications with requirements of very sensitive and precise force measurements.

In this paper we present our analysis of such a perturbed cognate mechanism which can be treated like a hypersensitive load cell. This new force measurement approach leads to a load cell whose force to displacement ratio is of the same order as the currently popular force sensor - the strain gauge. However, we observe that a new measure, i.e. the compliance at the point of measurement of the load cell to the applied load, is two orders of magnitude higher than conventional approaches.

Perturbed Cognate, Stiffness, Sensitivity, Load Cell, EigenPoint

### 1. Introduction

The cognate mechanism exhibits the unique stationary point feature because of which, the mechanism forms a structure whose links internally have 1 DOF. Conceptually, this property can be leveraged by utilizing the stationary point as a point of force loading to obtain a sensor with theoretically infinite stiffness if we can get the mechanism to exhibit different stable poses for different loading forces. We build upon our understanding of this theoretical problem and our knowledge about the applications of kinematic properties of mechanisms to ask if it is possible to synthesize a practically conceivable linkage based force sensor that can be used for haptic feedback and other applications with requirements of very sensitive and precise force measurements.

Current method of precise force measurement uses either a piezoelectric force sensor [1] or a capacitive force sensor [2] connected by an elastic member to the point of loading.

In this paper, we present the analysis of a perturbed cognate mechanism which can be treated like a hyper sensitive load cell. This new and theoretical load cell concept has a resultant stiffness comparable to the current state-of-art force sensor [1], the strain gauge. The sensitivity, i.e. the compliance at the point of measurement of the load cell, is two orders of magnitude larger than the strain gauge.

### 2. Cognate

A four bar mechanism consists of four elements: the crank, rocker, coupler and the ground frame. The coupler curve of a four bar mechanism is the path traced by a point on the coupler link. Using Roberts-Chebyshev Theorem [3, 4], it can be demonstrated that there are only three possible four bar mechanisms with same coupler curve. These mechanisms have a special name 'Cognates'.

This property is usually used to change fixed pivots of the already designed four bar mechanism. However, if these three cognate mechanism are joined to each other, a special property is observed in the resulting 10-link, 13-joint mechanism. If any two of the pivot joints are grounded, then for any motion given to the mechanism, third pivot point remains stationary with respect to the ground.

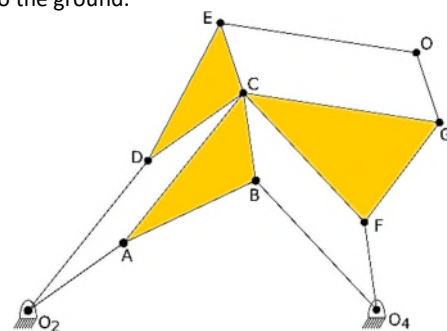


Figure 1: 10 link cognate mechanism

#### 2.1. Effect of springs at joints

Links with length to thickness ratio greater than 10 are used to introduce compliance. Such joints made using flexures could be represented as traditional hinges with torsional springs using pseudo-rigid body approaches. To simulate this effect in the flexure cognate, in this paper, we only add torsional springs of stiffness constant of  $15.8 \text{ N-mm rad}^{-1}$  at two joints, E and F, given that the gross mechanism size is  $40 \times 40 \text{ mm}^2$ . Theoretically, torsional springs could be introduced at all the joints.

The introduction of unbalanced springs in the mechanism joints leads to interesting properties that can be utilized for more applications. Examples are multi-stable mechanism synthesis (due to presence of multiple local minimas in the potential energy curve) and obtaining constant spring stiffness within a restricted range of motion. By definition of the cognate, if the point of application of force is O, it is stationary. Any link forces due to the torsional springs are simply superimposed on

the initial link forces. Hence, the mechanism is not able to differentiate between the various input forces by preferentially settling down at different equilibrium poses.

## 2.2. Possible perturbations

Perturbation in the mechanism (which may be later realised as a flexural joint based design [5]) can be broadly classified in two categories -

1. Coincident joints do not remain coincident.
2. Effective link lengths may change at various poses of the mechanism.

We assume that only type-1 perturbations are dominant in the mechanism. For analysis of the effect of such perturbations we assume that it occurs at joint C (Figure 1). This perturbation can be described by considering that the new joints  $C_1$  and  $C_2$  have a polar coordinates  $(r_1, \gamma_1)$  and  $(r_2, \gamma_2)$  respectively in the coordinate frame which is centered at C and whose X-axis is parallel to the Coupler.

$$C_1 = C + [r_1 \cos(\gamma_1 + \theta_3), r_1 \sin(\gamma_1 + \theta_3)]^T$$

$$C_2 = C + [r_2 \cos(\gamma_2 + \theta_3), r_2 \sin(\gamma_2 + \theta_3)]^T$$

## 3. Residual Moment Method

Forces and moments on each link are found analytically for an ideal cognate mechanism with torsional springs at the joints. This method showed that the moment on the coupler ABC is zero for any applied loading force. However upon distortion as above, the same approach shows that there is a residual moment on the coupler ABC, which is dependent on the crank angle.

The stability of the mechanism is defined in terms of the residual moment. This moment is defined about the instantaneous center of rotation of the coupler. We define the 'equilibrium position' of the distorted cognate mechanism to be that crank angle where the residual moment is zero.

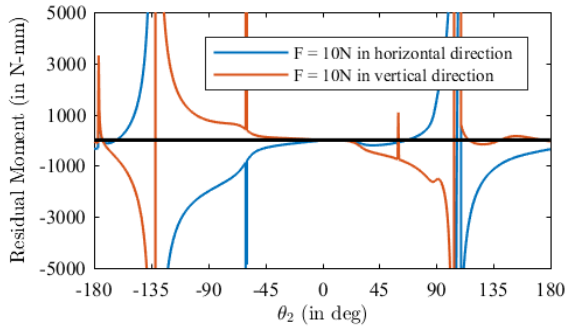


Figure 2: Residual moment variation for different loading condition

### 3.1. Equilibrium Points

The magnitude of the residual moment rises up to  $+\infty$  and loops back from  $-\infty$  at more than one value of the crank angle. This indicates that the distorted cognate mechanism is in a singular pose determined by one of the crank angles corresponding to indeterminate values of the residual moment. By singular pose, we refer to those crank angles of the mechanism at which adjacent links align with each other.

It is observed that the residual moment plot crosses the zero moment line several times for values of the crank angle between  $0^\circ$  and  $360^\circ$  (Figure 2). Some of the crossings however can be attributed to the asymptotic behaviour and may not be equilibrium positions in an equivalent physical model of the mechanism. For simplicity, we consider the crossing point closest to the starting crank angle to be the equilibrium point. Under that selection, Figure 3 shows the equilibrium positions of the perturbed mechanism corresponding to varying force direction and magnitude.

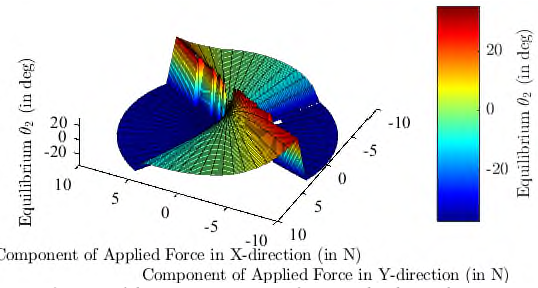


Figure 3: Equilibrium position of perturbed mechanism to varying force magnitude and direction

## 4. Adams Simulation

Adams is a multi-body dynamics simulation software [6]. A cognate model was created and simulated for the equilibrium position under variable force at previously stationary point of the cognate. The simulation agrees with the results obtained using the aforementioned method.

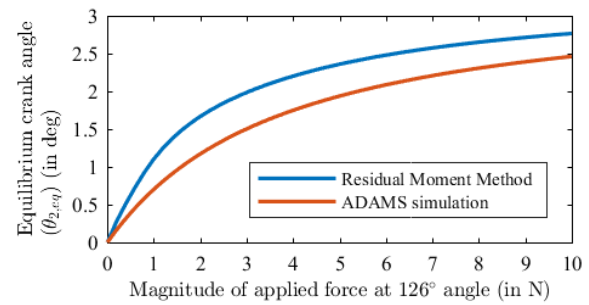


Figure 4: Comparison of the equilibrium position found using Adams and Residual Moment method

## 5. Conclusion

In our simulations, this new load cell design has a resultant length-normalized stiffness comparable to the currently predominant force sensor - the strain gauge ( $3.7 \times 10^3$  N v/s  $2.6 \times 10^3$  N respectively). The length-normalized sensitivity, i.e. the compliance at the point of measurement of the load cell, is two orders of magnitude larger than the strain gauge ( $3.5 \times 10^{-3}$  N $^{-1}$  v/s  $5.7 \times 10^{-5}$  N $^{-1}$  respectively). The proposed mechanism hence offer significantly higher movement (strain) at the point of measurement to that offered when using a strain gauge but offers comparable stiffness at the point of force application. A physical model of the mechanism is currently being designed for testing.

## 6. References

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